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11-1 Study Guide and Intervention

Simplifying Radical Expressions

Product Property of Square Roots The **Product Property of Square Roots** and prime factorization can be used to simplify expressions involving irrational square roots. When you simplify radical expressions with variables, use absolute value to ensure nonnegative results.

Product Property of Square Roots	For any numbers a and b, where $a \ge 0$ and $b \ge 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1 **Simplify $\sqrt{180}$.**

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$ Prime factorization of 180

$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 2 \cdot 3 \cdot \sqrt{5}$ Simplify.

$= 6\sqrt{5}$ Simplify.

Example 2 **Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.**

$\sqrt{120a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4}$

$= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2$

$= 2|a|b^2c^2\sqrt{30b}$

Exercises

Simplify.

1. $\sqrt{28}$ $2\sqrt{7}$
2. $\sqrt{68}$ $2\sqrt{17}$
3. $\sqrt{60}$ $2\sqrt{15}$
4. $\sqrt{75}$ $5\sqrt{3}$
5. $\sqrt{162}$ $9\sqrt{2}$
6. $\sqrt{3} \cdot \sqrt{6}$ $3\sqrt{2}$
7. $\sqrt{2} \cdot \sqrt{5}$ $\sqrt{10}$
8. $\sqrt{5} \cdot \sqrt{10}$ $5\sqrt{2}$
9. $\sqrt{4a^2}$ $2|a|$
10. $\sqrt{9x^4}$ $3x^2$
11. $\sqrt{300a^4}$ $10a^2\sqrt{3}$
12. $\sqrt{128c^6}$ $8|c^3|\sqrt{2}$
13. $4\sqrt{10} \cdot 3\sqrt{6}$ $24\sqrt{15}$
14. $\sqrt{3x^2} \cdot 3\sqrt{3x^4}$ $9|x^3|$
15. $\sqrt{20a^2b^4}$ $2|a|b^2\sqrt{5}$
16. $\sqrt{100x^3y}$ $10|x|\sqrt{xy}$
17. $\sqrt{24a^4b^2}$ $2a^2|b|\sqrt{6}$
18. $\sqrt{81x^4y^2}$ $9x^2|y|$
19. $\sqrt{150a^2b^2c}$ $5|ab|\sqrt{6c}$
20. $\sqrt{72a^6b^3c^2}$ $6|a^3bc|\sqrt{2b}$
21. $\sqrt{45x^2y^5z^8}$ $3|x|y^2z^4\sqrt{5y}$
22. $\sqrt{98x^4y^6z^2}$ $7x^2|y^3z|\sqrt{2}$

Lesson 11-1

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11-1 Study Guide and Intervention *(continued)*

Simplifying Radical Expressions

Quotient Property of Square Roots A fraction containing radicals is in simplest form if no radicals are left in the denominator. The **Quotient Property of Square Roots** and **rationalizing the denominator** can be used to simplify radical expressions that involve division. When you rationalize the denominator, you multiply the numerator and denominator by a radical expression that gives a rational number in the denominator.

Quotient Property of Square Roots	For any numbers a and b, where $a \ge 0$ and $b \ge 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example **Simplify $\sqrt{\frac{56}{45}}$.**

$\sqrt{\frac{56}{45}} = \sqrt{\frac{4 \cdot 14}{9 \cdot 5}}$

$= \frac{2 \cdot \sqrt{14}}{3 \cdot \sqrt{5}}$ Simplify the numerator and denominator.

$= \frac{2\sqrt{14}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominator.

$= \frac{2\sqrt{70}}{15}$ Product Property of Square Roots

Exercises

Simplify.

1. $\frac{\sqrt{9}}{\sqrt{18}}$ $\frac{\sqrt{2}}{2}$
2. $\frac{\sqrt{8}}{\sqrt{24}}$ $\frac{\sqrt{3}}{3}$
3. $\frac{\sqrt{100}}{\sqrt{121}}$ $\frac{10}{11}$
4. $\frac{\sqrt{75}}{\sqrt{3}}$ 5
5. $\frac{8\sqrt{2}}{2\sqrt{8}}$ 2
6. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{6}{5}}$ $\frac{2\sqrt{3}}{5}$
7. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{2}}$ $\frac{\sqrt{30}}{4}$
8. $\sqrt{\frac{5}{7}} \cdot \sqrt{\frac{2}{5}}$ $\frac{\sqrt{14}}{7}$
9. $\sqrt{\frac{3a^2}{10b^6}}$ $\frac{|a|\sqrt{30}}{10|b^3|}$
10. $\sqrt{\frac{x^6}{y^4}}$ $\frac{|x^3|}{y^2}$
11. $\sqrt{\frac{100a^4}{144b^8}}$ $\frac{5a^2}{6b^4}$
12. $\sqrt{\frac{75b^3c^6}{a^2}}$ $\frac{5|bc^3|\sqrt{3b}}{|a|}$
13. $\frac{\sqrt{4}}{3 - \sqrt{5}}$ $\frac{3 + \sqrt{5}}{2}$
14. $\frac{\sqrt{8}}{2 + \sqrt{3}}$ $4\sqrt{2} - 2\sqrt{6}$
15. $\frac{\sqrt{5}}{2 + \sqrt{5}}$ $5 - 2\sqrt{5}$
16. $\frac{\sqrt{8}}{2\sqrt{7} + 4\sqrt{10}}$ $\frac{4\sqrt{5} - \sqrt{14}}{33}$

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11-1 Skills Practice

Simplifying Radical Expressions

Simplify.

1. $\sqrt{28}$ **$2\sqrt{7}$**
2. $\sqrt{40}$ **$2\sqrt{10}$**
3. $\sqrt{72}$ **$6\sqrt{2}$**
4. $\sqrt{99}$ **$3\sqrt{11}$**
5. $\sqrt{2} \cdot \sqrt{10}$ **$2\sqrt{5}$**
6. $\sqrt{5} \cdot \sqrt{60}$ **$10\sqrt{3}$**
7. $3\sqrt{5} \cdot \sqrt{5}$ **15**
8. $\sqrt{6} \cdot 4\sqrt{24}$ **48**
9. $2\sqrt{3} \cdot 3\sqrt{15}$ **$18\sqrt{5}$**
10. $\sqrt{16b^4}$ **$4b^2$**
11. $\sqrt{81c^2d^4}$ **$9|c|d^2$**
12. $\sqrt{40x^4y^6}$ **$2x^2|y^3|\sqrt{10}$**
13. $\sqrt{75m^5n^2}$ **$5m^2|n|\sqrt{3m}$**
14. $\sqrt{\frac{5}{3}}$ **$\frac{\sqrt{15}}{3}$**
15. $\sqrt{\frac{1}{6}}$ **$\frac{\sqrt{6}}{6}$**
16. $\sqrt{\frac{6}{7}} \cdot \sqrt{\frac{1}{3}}$ **$\frac{\sqrt{14}}{7}$**
17. $\sqrt{\frac{q}{12}}$ **$\frac{\sqrt{3q}}{6}$**
18. $\sqrt{\frac{4h}{5}}$ **$\frac{2\sqrt{5h}}{5}$**
19. $\sqrt{\frac{12}{b^2}}$ **$\frac{2\sqrt{3}}{|b|}$**
20. $\sqrt{\frac{45}{4m^4}}$ **$\frac{3\sqrt{5}}{2m^2}$**
21. $\frac{2}{4 + \sqrt{5}}$ **$\frac{8 - 2\sqrt{5}}{11}$**
22. $\frac{3}{2 - \sqrt{3}}$ **$6 + 3\sqrt{3}$**
23. $\frac{5}{7 + \sqrt{7}}$ **$\frac{35 - 5\sqrt{7}}{42}$**
24. $\frac{4}{3 - \sqrt{2}}$ **$\frac{12 + 4\sqrt{2}}{7}$**

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11-1 Practice (Average)

Simplifying Radical Expressions

Simplify.

1. $\sqrt{24}$ **$2\sqrt{6}$**
2. $\sqrt{60}$ **$2\sqrt{15}$**
3. $\sqrt{108}$ **$6\sqrt{3}$**
4. $\sqrt{8} \cdot \sqrt{6}$ **$4\sqrt{3}$**
5. $\sqrt{7} \cdot \sqrt{14}$ **$7\sqrt{2}$**
6. $3\sqrt{12} \cdot 5\sqrt{6}$ **$90\sqrt{2}$**
7. $4\sqrt{3} \cdot 3\sqrt{18}$ **$36\sqrt{6}$**
8. $\sqrt{27su^3}$ **$3|u|\sqrt{3su}$**
9. $\sqrt{50p^5}$ **$5p^2\sqrt{2p}$**
10. $\sqrt{108x^6y^4z^5}$ **$6|x^3|y^2z^2\sqrt{3z}$**
11. $\sqrt{56m^2n^4o^5}$ **$2|m|n^2o^2\sqrt{14o}$**
12. $\frac{\sqrt{8}}{\sqrt{6}}$ **$\frac{2\sqrt{3}}{3}$**
13. $\sqrt{\frac{2}{10}}$ **$\frac{\sqrt{5}}{5}$**
14. $\sqrt{\frac{5}{32}}$ **$\frac{\sqrt{10}}{8}$**
15. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{4}{5}}$ **$\frac{\sqrt{15}}{5}$**
16. $\sqrt{\frac{1}{7}} \cdot \sqrt{\frac{7}{11}}$ **$\frac{\sqrt{11}}{11}$**
17. $\frac{\sqrt{3k}}{\sqrt{8}}$ **$\frac{\sqrt{6k}}{4}$**
18. $\sqrt{\frac{18}{x^3}}$ **$\frac{3\sqrt{2x}}{x^2}$**
19. $\sqrt{\frac{4y}{3y^2}}$ **$\frac{2\sqrt{3y}}{3|y|}$**
20. $\sqrt{\frac{9ab}{4ab^4}}$ **$\frac{3\sqrt{b}}{2b^2}$**
21. $\frac{3}{5 - \sqrt{2}}$ **$\frac{15 + 3\sqrt{2}}{23}$**
22. $\frac{8}{3 + \sqrt{3}}$ **$\frac{12 - 4\sqrt{3}}{3}$**
23. $\frac{5}{\sqrt{7} + \sqrt{3}}$ **$\frac{5\sqrt{7} - 5\sqrt{3}}{4}$**
24. $\frac{3\sqrt{7}}{-1 - \sqrt{27}}$ **$\frac{3\sqrt{7} - 9\sqrt{21}}{26}$**

25. **SKY DIVING** When a skydiver jumps from an airplane, the time t it takes to free fall a given distance can be estimated by the formula $t = \sqrt{\frac{2s}{9.8}}$, where t is in seconds and s is in meters. If Julie jumps from an airplane, how long will it take her to free fall 750 meters? **about 12.4 s**

METEOROLOGY For Exercises 26 and 27, use the following information.

To estimate how long a thunderstorm will last, meteorologists can use the formula $t = \sqrt{\frac{d^3}{216}}$, where t is the time in hours and d is the diameter of the storm in miles.

26. A thunderstorm is 8 miles in diameter. Estimate how long the storm will last. Give your answer in simplified form and as a decimal. **$\frac{8\sqrt{3}}{9}$ h ≈ 1.5 h**

27. Will a thunderstorm twice this diameter last twice as long? Explain. **No; it will last about 4.4 h, or nearly 3 times as long.**

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11-1 Reading to Learn Mathematics

Simplifying Radical Expressions

Pre-Activity **How are radical expressions used in space exploration?**

Read the introduction to Lesson 11-1 at the top of page 586 in your textbook.

Suppose you want to calculate the escape velocity for a spacecraft taking off from the planet Mars. When you substitute numbers in the formula, which number is sure to be the same as in the calculation for the escape velocity for a spacecraft taking off from Earth? **the value of *G***

Reading the Lesson

1. a. How can you tell that the radical expression $\sqrt{28x^2y^4}$ is not in simplest form?

The radicand contains perfect square factors other than 1.

b. To simplify $\sqrt{28x^2y^4}$, you first find the **prime factorization** of $28x^2y^4$.

You then apply the **Product Property of Square Roots**. In this case, $\sqrt{4 \cdot 7 \cdot x^2 \cdot y^4}$ is equal to the product $\sqrt{4} \cdot \sqrt{7} \cdot \sqrt{x^2} \cdot \sqrt{y^4}$. You can simplify again to get a final answer of $2|x|y^2\sqrt{7}$.

2. Why is it correct to write $\sqrt{y^4} = y^2$, with no absolute value sign, but not correct to write $\sqrt{x^2} = x$?

Sample answer: The square of y^2 is y^4 and the expressions $\sqrt{y^4}$ and y^2 both represent positive numbers for all values of y. Although it is true that the square of x is x^2, when x is less than 0, $\sqrt{x^2}$ represents a positive quantity and x represents a negative quantity.

3. What method would you use to simplify $\frac{\sqrt{12t}}{\sqrt{15}}$?

rationalizing the denominator

4. What should you do to write the conjugate of a binomial of the form $a\sqrt{b} + c\sqrt{d}$? To write the conjugate of a binomial of the form $a\sqrt{b} - c\sqrt{d}$?

Change the plus sign to a minus sign; change the minus sign to a plus sign.

Helping You Remember

5. What should you remember to check for when you want to determine if a radical expression is in simplest form?

Sample answer: Check radicands for perfect squares and fractions, and check fractions for radicals in the denominator.

Lesson 11-1

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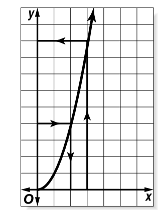
11-1 Enrichment

Squares and Square Roots From a Graph



The graph of $y = x^2$ can be used to find the squares and square roots of numbers.

To find the square of 3, locate 3 on the x-axis. Then find its corresponding value on the y-axis.

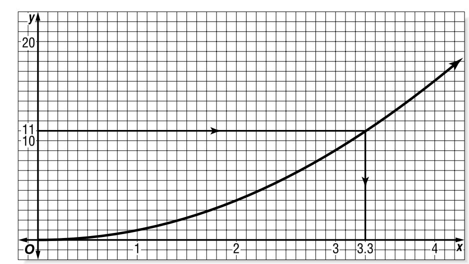
The arrows show that $3^2 = 9$.

To find the square root of 4, first locate 4 on the y-axis. Then find its corresponding value on the x-axis. Following the arrows on the graph, you can see that $\sqrt{4} = 2$.

A small part of the graph at $y = x^2$ is shown below. A 1:10 ratio for unit length on the y-axis to unit length on the x-axis is used.

Example **Find $\sqrt{11}$.**

The arrows show that $\sqrt{11} = 3.3$ to the nearest tenth.



Use the graph above to find each of the following to the nearest whole number.

1. 1.5^2 **2**

2. 2.7^2 **7**

3. 0.9^2 **1**

4. 3.6^2 **13**

5. 4.2^2 **18**

6. 3.9^2 **15**

Use the graph above to find each of the following to the nearest tenth.

7. $\sqrt{15}$ **3.9**

8. $\sqrt{8}$ **2.8**

9. $\sqrt{3}$ **1.7**

10. $\sqrt{5}$ **2.2**

11. $\sqrt{14}$ **3.7**

12. $\sqrt{17}$ **4.1**

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11-2 Study Guide and Intervention

Operations with Radical Expressions

Add and Subtract Radical Expressions When adding or subtracting radical expressions, use the Associative and Distributive Properties to simplify the expressions. If radical expressions are not in simplest form, simplify them.

Example 1 **Simplify $10\sqrt{6} - 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{6}$.**

$10\sqrt{6} - 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{6} = (10 - 4)\sqrt{6} + (-5 + 6)\sqrt{3}$ Associative and Distributive Properties

$= 6\sqrt{6} + \sqrt{3}$ Simplify.

Example 2 **Simplify $3\sqrt{12} + 5\sqrt{75}$.**

$3\sqrt{12} + 5\sqrt{75} = 3\sqrt{2^2 \cdot 3} + 5\sqrt{5^2 \cdot 3}$ Simplify.

$= 3 \cdot 2\sqrt{3} + 5 \cdot 5\sqrt{3}$ Simplify.

$= 6\sqrt{3} + 25\sqrt{3}$ Simplify.

$= 31\sqrt{3}$ Distributive Property

Exercises

Simplify each expression.

1. $2\sqrt{5} + 4\sqrt{5}$ $6\sqrt{5}$

2. $\sqrt{6} - 4\sqrt{6}$ $-3\sqrt{6}$

3. $\sqrt{8} - \sqrt{2}$ $\sqrt{2}$

4. $3\sqrt{75} + 2\sqrt{5}$ $15\sqrt{3} + 2\sqrt{5}$

5. $\sqrt{20} + 2\sqrt{5} - 3\sqrt{5}$ $\sqrt{5}$

6. $2\sqrt{3} + \sqrt{6} - 5\sqrt{3}$ $-3\sqrt{3} + \sqrt{6}$

7. $\sqrt{12} + 2\sqrt{3} - 5\sqrt{3}$ $-\sqrt{3}$

8. $3\sqrt{6} + 3\sqrt{2} - \sqrt{50} + \sqrt{24}$ $5\sqrt{6} - 2\sqrt{2}$

9. $\sqrt{8a} - \sqrt{2a} + 5\sqrt{2a}$ $6\sqrt{2a}$

10. $\sqrt{54} + \sqrt{24}$ $5\sqrt{6}$

11. $\sqrt{3} + \sqrt{\frac{1}{3}}$ $\frac{4\sqrt{3}}{3}$

12. $\sqrt{12} + \sqrt{\frac{1}{3}}$ $\frac{7\sqrt{3}}{3}$

13. $\sqrt{54} - \sqrt{\frac{1}{6}}$ $\frac{17\sqrt{6}}{6}$

14. $\sqrt{80} - \sqrt{20} + \sqrt{180}$ $8\sqrt{5}$

15. $\sqrt{50} + \sqrt{18} - \sqrt{75} + \sqrt{27}$ $8\sqrt{2} - 2\sqrt{3}$

16. $2\sqrt{3} - 4\sqrt{45} + 2\sqrt{\frac{1}{3}}$ $\frac{8\sqrt{3}}{3} - 12\sqrt{5}$

17. $\sqrt{125} - 2\sqrt{\frac{1}{5}} + \sqrt{\frac{1}{3}}$ $\frac{23\sqrt{5}}{5} + \frac{\sqrt{3}}{3}$

18. $\sqrt{\frac{2}{3}} + 3\sqrt{3} - 4\sqrt{\frac{1}{12}}$ $\frac{\sqrt{6}}{3} + \frac{28\sqrt{3}}{12}$

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11-2 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Multiply Radical Expressions Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example **Multiply $(3\sqrt{2} - 2\sqrt{5})(4\sqrt{20} + \sqrt{8})$.**

Use the FOIL method.

$(3\sqrt{2} - 2\sqrt{5})(4\sqrt{20} + \sqrt{8}) = (3\sqrt{2})(4\sqrt{20}) + (3\sqrt{2})(\sqrt{8}) + (-2\sqrt{5})(4\sqrt{20}) + (-2\sqrt{5})(\sqrt{8})$

$= 12\sqrt{40} + 3\sqrt{16} - 8\sqrt{100} - 2\sqrt{40}$ Multiply.

$= 12\sqrt{2^2 \cdot 10} + 3 \cdot 4 - 8 \cdot 10 - 2\sqrt{2^2 \cdot 10}$ Simplify.

$= 24\sqrt{10} + 12 - 80 - 4\sqrt{10}$ Simplify.

$= 20\sqrt{10} - 68$ Combine like terms.

Exercises

Find each product.

1. $2(\sqrt{3} + 4\sqrt{5})$
$2\sqrt{3} + 8\sqrt{5}$

2. $\sqrt{6}(\sqrt{3} - 2\sqrt{6})$
$3\sqrt{2} - 12$

3. $\sqrt{5}(\sqrt{5} - \sqrt{2})$
$5 - \sqrt{10}$

4. $\sqrt{2}(3\sqrt{7} + 2\sqrt{5})$
$3\sqrt{14} + 2\sqrt{10}$

5. $(2 - 4\sqrt{2})(2 + 4\sqrt{2})$
-28

6. $(3 + \sqrt{6})^2$
$15 + 6\sqrt{6}$

7. $(2 - 2\sqrt{5})^2$
$24 - 8\sqrt{5}$

8. $3\sqrt{2}(\sqrt{8} + \sqrt{24})$
$12 + 12\sqrt{3}$

9. $\sqrt{8}(\sqrt{2} + 5\sqrt{8})$
44

10. $(\sqrt{5} - 3\sqrt{2})(\sqrt{5} + 3\sqrt{2})$
-13

11. $(\sqrt{3} + \sqrt{6})^2$
$9 + 6\sqrt{2}$

12. $(\sqrt{2} - 2\sqrt{3})^2$
$14 - 4\sqrt{6}$

13. $(\sqrt{5} - \sqrt{2})(\sqrt{2} + \sqrt{6})$
$\sqrt{10} - 2 + \sqrt{30} - 2\sqrt{3}$

14. $(\sqrt{8} - \sqrt{2})(\sqrt{3} + \sqrt{6})$
$\sqrt{6} + 2\sqrt{3}$

15. $(\sqrt{5} - \sqrt{18})(7\sqrt{5} + \sqrt{3})$
$35 + \sqrt{15} - 2\sqrt{10} - 3\sqrt{6}$

16. $(2\sqrt{3} - \sqrt{45})(\sqrt{12} + 2\sqrt{6})$
$12 - 6\sqrt{15} + 12\sqrt{2} - 6\sqrt{30}$

17. $(2\sqrt{5} - 2\sqrt{3})(\sqrt{10} + \sqrt{6})$
$4\sqrt{2}$

18. $(\sqrt{2} + 3\sqrt{3})(\sqrt{12} - 4\sqrt{8})$
$2 - 22\sqrt{6}$

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11-2 Skills Practice

Operations with Radical Expressions

Simplify each expression.

1. $7\sqrt{7} - 2\sqrt{7}$ **$5\sqrt{7}$**

2. $3\sqrt{13} + 7\sqrt{13}$ **$10\sqrt{13}$**

3. $6\sqrt{5} - 2\sqrt{5} + 8\sqrt{5}$ **$12\sqrt{5}$**

4. $\sqrt{15} + 8\sqrt{15} - 12\sqrt{15}$ **$-3\sqrt{15}$**

5. $12\sqrt{c} - 9\sqrt{c}$ **$3\sqrt{c}$**

6. $9\sqrt{6a} - 11\sqrt{6a} + 4\sqrt{6a}$ **$2\sqrt{6a}$**

7. $\sqrt{44} - \sqrt{11}$ **$\sqrt{11}$**

8. $\sqrt{28} + \sqrt{63}$ **$5\sqrt{7}$**

9. $4\sqrt{3} + 2\sqrt{12}$ **$8\sqrt{3}$**

10. $8\sqrt{54} - 4\sqrt{6}$ **$20\sqrt{6}$**

11. $\sqrt{27} + \sqrt{48} + \sqrt{12}$ **$9\sqrt{3}$**

12. $\sqrt{72} + \sqrt{50} - \sqrt{8}$ **$9\sqrt{2}$**

13. $\sqrt{180} - 5\sqrt{5} + \sqrt{20}$ **$3\sqrt{5}$**

14. $2\sqrt{24} + 4\sqrt{54} + 5\sqrt{96}$ **$36\sqrt{6}$**

15. $5\sqrt{8} + 2\sqrt{20} - \sqrt{8}$
$8\sqrt{2} + 4\sqrt{5}$

16. $2\sqrt{13} + 4\sqrt{2} - 5\sqrt{13} + \sqrt{2}$
$-3\sqrt{13} + 5\sqrt{2}$

Find each product.

17. $\sqrt{2}(\sqrt{8} + \sqrt{6})$ **$4 + 2\sqrt{3}$**

18. $\sqrt{5}(\sqrt{10} - \sqrt{3})$ **$5\sqrt{2} - \sqrt{15}$**

19. $\sqrt{6}(3\sqrt{2} - 2\sqrt{3})$ **$6\sqrt{3} - 6\sqrt{2}$**

20. $3\sqrt{3}(2\sqrt{6} + 4\sqrt{10})$ **$18\sqrt{2} + 12\sqrt{30}$**

21. $(4 + \sqrt{3})(4 - \sqrt{3})$ **13**

22. $(2 - \sqrt{6})^2$ **$10 - 4\sqrt{6}$**

23. $(\sqrt{8} + \sqrt{2})(\sqrt{5} + \sqrt{3})$
$3\sqrt{10} + 3\sqrt{6}$

24. $(\sqrt{6} + 4\sqrt{5})(4\sqrt{3} - \sqrt{10})$
$-8\sqrt{2} + 14\sqrt{15}$

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11-2 Practice (Average)

Operations with Radical Expressions

Simplify each expression.

1. $8\sqrt{30} - 4\sqrt{30}$ **$4\sqrt{30}$**

2. $2\sqrt{5} + 7\sqrt{5} - 5\sqrt{5}$ **$4\sqrt{5}$**

3. $7\sqrt{13x} - 14\sqrt{13x} + 2\sqrt{13x}$ **$-5\sqrt{13x}$**

4. $2\sqrt{45} + 4\sqrt{20}$ **$14\sqrt{5}$**

5. $\sqrt{40} - \sqrt{10} + \sqrt{90}$ **$4\sqrt{10}$**

6. $2\sqrt{32} + 3\sqrt{50} - 3\sqrt{18}$ **$14\sqrt{2}$**

7. $\sqrt{27} + \sqrt{18} + \sqrt{300}$ **$3\sqrt{2} + 13\sqrt{3}$**

8. $5\sqrt{8} + 3\sqrt{20} - \sqrt{32}$ **$6\sqrt{2} + 6\sqrt{5}$**

9. $\sqrt{14} - \sqrt{\frac{2}{7}}$ **$\frac{6\sqrt{14}}{7}$**

10. $\sqrt{50} + \sqrt{32} - \sqrt{\frac{1}{2}}$ **$\frac{17\sqrt{2}}{2}$**

11. $5\sqrt{19} + 4\sqrt{28} - 8\sqrt{19} + \sqrt{63}$
$-3\sqrt{19} + 11\sqrt{7}$

12. $3\sqrt{10} + \sqrt{75} - 2\sqrt{40} - 4\sqrt{12}$
$-\sqrt{10} - 3\sqrt{3}$

Find each product.

13. $\sqrt{6}(\sqrt{10} + \sqrt{15})$ **$2\sqrt{15} + 3\sqrt{10}$**

14. $\sqrt{5}(5\sqrt{2} - 4\sqrt{8})$ **$-3\sqrt{10}$**

15. $2\sqrt{7}(3\sqrt{12} + 5\sqrt{8})$ **$12\sqrt{21} + 20\sqrt{14}$**

16. $(5 - \sqrt{15})^2$ **$40 - 10\sqrt{15}$**

17. $(\sqrt{10} + \sqrt{6})(\sqrt{30} - \sqrt{18})$ **$4\sqrt{3}$**

18. $(\sqrt{8} + \sqrt{12})(\sqrt{48} + \sqrt{18})$ **$36 + 14\sqrt{6}$**

19. $(\sqrt{2} + 2\sqrt{8})(3\sqrt{6} - \sqrt{5})$
$30\sqrt{3} - 5\sqrt{10}$

20. $(4\sqrt{3} - 2\sqrt{5})(3\sqrt{10} + 5\sqrt{6})$
$2\sqrt{30} + 30\sqrt{2}$

SOUND For Exercises 21 and 22, use the following information.
The speed of sound V in meters per second near Earth's surface is given by $V = 20\sqrt{t + 273}$, where t is the surface temperature in degrees Celsius.

21. What is the speed of sound near Earth's surface at 15°C and at 2°C in simplest form?
$240\sqrt{2}$ m/s, $100\sqrt{11}$ m/s

22. How much faster is the speed of sound at 15°C than at 2°C?
$240\sqrt{2} - 100\sqrt{11} \approx 7.75$ m/s

GEOMETRY For Exercises 23 and 24, use the following information.
A rectangle is $5\sqrt{7} + 2\sqrt{3}$ centimeters long and $6\sqrt{7} - 3\sqrt{3}$ centimeters wide.

23. Find the perimeter of the rectangle in simplest form. **$22\sqrt{7} - 2\sqrt{3}$ cm**

24. Find the area of the rectangle in simplest form. **$192 - 3\sqrt{21}$ cm^2**

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11-2 Reading to Learn Mathematics

Operations with Radical Expressions

Pre-Activity **How can you use radical expressions to determine how far a person can see?**

Read the introduction to Lesson 11-2 at the top of page 593 in your textbook.

Suppose you substitute the heights of the Sears Tower and the Empire State Building into the formula to find how far you can see from atop each building. What operation should you then use to determine how much farther you can see from the Sears Tower than from the Empire State Building?

subtraction

Reading the Lesson

1. Indicate whether the following expressions are in simplest form. Explain your answer.

a. $6\sqrt{3} - \sqrt{12}$

No; 12 can be simplified to $\sqrt{2^2} \cdot \sqrt{3}$ or $2\sqrt{3}$.

b. $12\sqrt{6} + 7\sqrt{10}$

Yes; both the addends are radical expressions in simplest form, the radicands are different, and there are no common factors.

2. Below the words **F**irst terms, **O**uter terms, **I**nner terms, and **L**ast terms, write the products you would use to simplify the expression $(2\sqrt{15} + 3\sqrt{15})(6\sqrt{3} - 5\sqrt{2})$.

First terms		Outer terms		Inner terms		Last terms
$(2\sqrt{15})(6\sqrt{3})$	+	$(2\sqrt{15})(-5\sqrt{2})$	+	$(3\sqrt{5})(6\sqrt{3})$	+	$(3\sqrt{5})(-5\sqrt{2})$

Helping You Remember

3. How can you use what you know about adding and subtracting monomials to help you remember how to add and subtract radical expressions?

Sample answer: Check that the addends have been simplified. Next, group addends that involve like radicals. Then use the Distributive Property to combine the addends that involve like radicals.

Lesson 11-2

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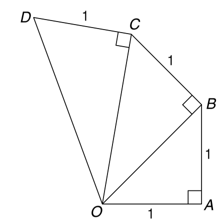
11-2 Enrichment

The Wheel of Theodorus

The Greek mathematicians were intrigued by problems of representing different numbers and expressions using geometric constructions.

Theodorus, a Greek philosopher who lived about 425 B.C., is said to have discovered a way to construct the sequence $\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots$.

The beginning of his construction is shown. You start with an isosceles right triangle with sides 1 unit long.



Use the figure above. Write each length as a radical expression in simplest form.

1. line segment AO **$\sqrt{1}$**

2. line segment BO **$\sqrt{2}$**

3. line segment CO **$\sqrt{3}$**

4. line segment DO **$\sqrt{4}$**

5. Describe how each new triangle is added to the figure. **Draw a new side of length 1 at right angles to the last hypotenuse. Then draw the new hypotenuse.**

6. The length of the hypotenuse of the first triangle is $\sqrt{2}$. For the second triangle, the length is $\sqrt{3}$. Write an expression for the length of the hypotenuse of the nth triangle.

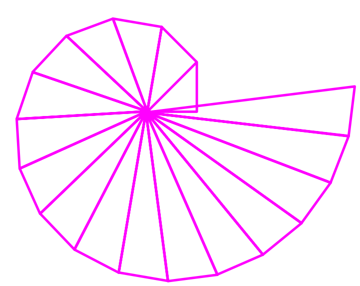
$\sqrt{n+1}$

7. Show that the method of construction will always produce the next number in the sequence. (*Hint:* Find an expression for the hypotenuse of the $(n + 1)$th triangle.)

$\sqrt{(\sqrt{n})^2 + (1)^2}$ or $\sqrt{n+1}$

8. In the space below, construct a Wheel of Theodorus. Start with a line segment 1 centimeter long. When does the Wheel start to overlap?

after length $\sqrt{18}$



Answers

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11-3 Study Guide and Intervention

Radical Equations

Radical Equations Equations containing radicals with variables in the radicand are called **radical equations**. These can be solved by first using the following steps.

Step 1	Isolate the radical on one side of the equation.
Step 2	Square each side of the equation to eliminate the radical.

Example 1 **Solve $16 = \frac{\sqrt{x}}{2}$ for x.**

$16 = \frac{\sqrt{x}}{2}$ Original equation

$2(16) = 2\left(\frac{\sqrt{x}}{2}\right)$ Multiply each side by 2.

$32 = \sqrt{x}$ Simplify.

$(32)^2 = (\sqrt{x})^2$ Square each side.

$1024 = x$ Simplify.

The solution is 1024, which checks in the original equation.

Example 2 **Solve $\sqrt{4x - 7} + 2 = 7$.**

$\sqrt{4x - 7} + 2 = 7$ Original equation

$\sqrt{4x - 7} + 2 - 2 = 7 - 2$ Subtract 2 from each side.

$\sqrt{4x - 7} = 5$ Simplify.

$(\sqrt{4x - 7})^2 = 5^2$ Square each side.

$4x - 7 = 25$ Simplify.

$4x - 7 + 7 = 25 + 7$ Add 7 to each side.

$4x = 32$ Simplify.

$x = 8$ Divide each side by 4.

The solution is 8, which checks in the original equation.

Exercises

Solve each equation. Check your solution.

1. $\sqrt{a} = 8$ **64**
2. $\sqrt{a} + 6 = 32$ **676**
3. $2\sqrt{x} = 8$ **16**
4. $7 = \sqrt{26 - n}$ **−23**
5. $\sqrt{-a} = 6$ **−36**
6. $\sqrt{3r^2} = 3$ **$\pm\sqrt{3}$**
7. $2\sqrt{3} = \sqrt{y}$ **12**
8. $2\sqrt{3a} - 2 = 7$ **$6\frac{3}{4}$**
9. $\sqrt{x - 4} = 6$ **40**
10. $\sqrt{2c + 3} = 5$ **11**
11. $\sqrt{3b - 2} + 19 = 24$ **9**
12. $\sqrt{4x - 1} = 3$ **$\frac{5}{2}$**
13. $\sqrt{3r + 2} = 2\sqrt{3}$ **$\frac{10}{3}$**
14. $\sqrt{\frac{x}{2}} = \frac{1}{2}$ **$\frac{1}{2}$**
15. $\sqrt{\frac{x}{8}} = 4$ **128**
16. $\sqrt{6x^2 + 5x} = 2$ **$\frac{1}{2}, -\frac{4}{3}$**
17. $\sqrt{\frac{x}{3}} + 6 = 8$ **12**
18. $2\sqrt{\frac{3x}{5}} + 3 = 11$ **$26\frac{2}{3}$**

Lesson 11-3

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11-3 Study Guide and Intervention *(continued)*

Radical Equations

Extraneous Solutions To solve a radical equation with a variable on both sides, you need to square each side of the equation. Squaring each side of an equation sometimes produces **extraneous solutions**, or solutions that are not solutions of the original equation. Therefore, it is very important that you check each solution.

Example **Solve $\sqrt{x + 3} = x - 3$.**

$\sqrt{x + 3} = x - 3$ Original equation

$(\sqrt{x + 3})^2 = (x - 3)^2$ Square each side.

$x + 3 = x^2 - 6x + 9$ Simplify.

$0 = x^2 - 7x + 6$ Subtract x and 3 from each side.

$0 = (x - 1)(x - 6)$ Factor.

$x - 1 = 0$ or $x - 6 = 0$ Zero Product Property

$x = 1$ $\quad x = 6$ Solve.

CHECK

$\sqrt{x + 3} = x - 3$ $\qquad \sqrt{x + 3} = x - 3$

$\sqrt{1 + 3} \stackrel{?}{=} 1 - 3$ $\qquad \sqrt{6 + 3} \stackrel{?}{=} 6 - 3$

$\sqrt{4} \stackrel{?}{=} -2$ $\qquad \sqrt{9} \stackrel{?}{=} 3$

$2 \neq -2$ $\qquad 3 = 3$ ✓

Since $x = 1$ does not satisfy the original equation, $x = 6$ is the only solution.

Exercises

Solve each equation. Check your solution.

1. $\sqrt{a} = a$ **0, 1**
2. $\sqrt{a + 6} = a$ **3**
3. $2\sqrt{x} = x$ **0, 4**
4. $n = \sqrt{2 - n}$ **1**
5. $\sqrt{-a} = a$ **0**
6. $\sqrt{10 - 6k} + 3 = k$ **∅**
7. $\sqrt{y - 1} = y - 1$ **1, 2**
8. $\sqrt{3a - 2} = a$ **1, 2**
9. $\sqrt{x + 2} = x$ **2**
10. $\sqrt{2c + 5} = c - 5$ **10**
11. $\sqrt{3b + 6} = b + 2$ **1**
12. $\sqrt{4x - 4} = x$ **2**
13. $r + \sqrt{2 - r} = 2$ **1, 2**
14. $\sqrt{x^2 + 10x} = x + 4$ **8**
15. $-2\sqrt{\frac{x}{8}} = 15$ **∅**
16. $\sqrt{6x^2 - 4x} = x + 2$ **$-\frac{2}{5}, 2$**
17. $\sqrt{2y^2 - 64} = y$ **8**
18. $\sqrt{3x^2 + 12x + 1} = x + 5$ **−4, 3**

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11-3 Skills Practice

Radical Equations

Solve each equation. Check your solution.

1. $\sqrt{f} = 7$ **49**
2. $\sqrt{-x} = 5$ **−25**
3. $\sqrt{5p} = 10$ **20**
4. $\sqrt{4y} = 6$ **9**
5. $2\sqrt{2} = \sqrt{u}$ **8**
6. $3\sqrt{5} = \sqrt{-n}$ **−45**
7. $\sqrt{g} - 6 = 3$ **81**
8. $\sqrt{5a} + 2 = 0$ **no solution**
9. $\sqrt{2c - 1} = 5$ **13**
10. $\sqrt{3k - 2} = 4$ **6**
11. $\sqrt{x + 4} - 2 = 1$ **5**
12. $\sqrt{4x - 4} - 4 = 0$ **5**
13. $\frac{\sqrt{d}}{3} = 4$ **144**
14. $\sqrt{\frac{m}{3}} = 3$ **27**
15. $x = \sqrt{x + 2}$ **2**
16. $d = \sqrt{12 - d}$ **3**
17. $\sqrt{6x - 9} = x$ **3**
18. $\sqrt{6p - 8} = p$ **2, 4**
19. $\sqrt{x + 5} = x - 1$ **4**
20. $\sqrt{8 - c} = c - 8$ **8**
21. $\sqrt{r - 3} + 5 = r$ **7**
22. $\sqrt{y - 1} + 3 = y$ **5**
23. $\sqrt{5n + 4} = n + 2$ **1, 0**
24. $\sqrt{3z - 6} = z - 2$ **5, 2**

Lesson 11-3

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11-3 Practice (Average)

Radical Equations

Solve each equation. Check your solution.

1. $\sqrt{-b} = 8$ **−64**
2. $4\sqrt{3} = \sqrt{x}$ **48**
3. $2\sqrt{4c} + 3 = 11$ **4**
4. $6 - \sqrt{2y} = -2$ **32**
5. $\sqrt{k + 2} - 3 = 7$ **98**
6. $\sqrt{m - 5} = 4\sqrt{3}$ **53**
7. $\sqrt{6t + 12} = 8\sqrt{6}$ **62**
8. $\sqrt{3j - 11} + 2 = 9$ **20**
9. $\sqrt{2x + 15} + 5 = 18$ **77**
10. $\sqrt{\frac{3s}{5}} - 4 = 2$ **60**
11. $6\sqrt{\frac{3x}{3}} - 3 = 0$ **$\frac{1}{4}$**
12. $6 + \sqrt{\frac{5r}{6}} = -2$ **no solution**
13. $y = \sqrt{y + 6}$ **3**
14. $\sqrt{15 - 2x} = x$ **3**
15. $\sqrt{w + 4} = w + 4$ **−4, −3**
16. $\sqrt{17 - k} = k - 5$ **8**
17. $\sqrt{5m - 16} = m - 2$ **4, 5**
18. $\sqrt{24 + 8q} = q + 3$ **−3, 5**
19. $\sqrt{4s + 17} - s - 3 = 0$ **2**
20. $4 - \sqrt{3m + 28} = m$ **−1**
21. $\sqrt{10p + 61} - 7 = p$ **−6, 2**
22. $\sqrt{2x^2 - 9} = x$ **3**

ELECTRICITY For Exercises 23 and 24, use the following information.
The voltage V in a circuit is given by $V = \sqrt{PR}$, where P is the power in watts and R is the resistance in ohms.

23. If the voltage in a circuit is 120 volts and the circuit produces 1500 watts of power, what is the resistance in the circuit? **9.6 ohms**
24. Suppose an electrician designs a circuit with 110 volts and a resistance of 10 ohms. How much power will the circuit produce? **1210 watts**

FREE FALL For Exercises 25 and 26, use the following information.
Assuming no air resistance, the time t in seconds that it takes an object to fall h feet can be determined by the equation $t = \frac{\sqrt{h}}{4}$.

25. If a skydiver jumps from an airplane and free falls for 10 seconds before opening the parachute, how many feet does the skydiver fall? **1600 ft**
26. Suppose a second skydiver jumps and free falls for 6 seconds. How many feet does the second skydiver fall? **576 ft**

Answers

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11-3 Reading to Learn Mathematics

Radical Equations

Pre-Activity **How are radical equations used to find free-fall times?**

Read the introduction to Lesson 11-3 at the top of page 598 in your textbook.

How can you isolate $\sqrt{h}$ on one side of the equation?

Multiply each side of the equation by 4.

Reading the Lesson

1. To solve a radical equation, you first isolate the radical on one side of the equation. Why do you then square each side of the equation?

to eliminate the radical

2. a. Provide the reason for each step in the solution of the given radical equation.

$\sqrt{5x-1} - 4 = x - 3$	Original equation
$\sqrt{5x-1} = x + 1$	**Add 4 to each side.**
$(\sqrt{5x-1})^2 = (x+1)^2$	**Square each side.**
$5x - 1 = x^2 + 2x + 1$	**Simplify.**
$0 = x^2 - 3x + 2$	**Subtract 5*x* and add 1 to each side.**
$0 = (x-1)(x-2)$	**Factor.**
$x - 1 = 0$ or $x - 2 = 0$	**Zero Product Property**
$x = 1$ $\quad x = 2$	**Solve.**

b. To be sure that 1 and 2 are the correct solutions, into which equation should you substitute to check? **the original equation**

3. a. How do you determine whether an equation has an extraneous solution?

Substitute the solution(s) into the original equation. If a solution does not satisfy the original equation, then it is an extraneous solution.

b. Is it necessary to check all solutions to eliminate extraneous solutions? Explain.

Yes; since you square each side of a radical equation, and squaring each side can sometimes produce an extraneous solution, you need to check all solutions. The only way to be sure that a solution is not extraneous is to check it in the original equation.

Helping You Remember

4. How can you use the letters ISC to remember the three steps in solving a radical equation?

Sample answer: Isolate the radical on one side of the equation, Square each side to eliminate the radical, and Check for extraneous solutions.

Lesson 11-3

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11-3 Enrichment

Special Polynomial Products

Sometimes the product of two polynomials can be found readily with the use of one of the special products of binomials.

For example, you can find the square of a trinomial by recalling the square of a binomial.

Example 1 **Find $(x + y + z)^2$.**

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2$$
$$[(x+y)+z]^2 = (x+y)^2 + 2(x+y)z + z^2$$
$$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$$

Example 2 **Find $(3t + x + 1)(3t - x - 1)$.**

(*Hint:* $(3t + x + 1)(3t - x - 1)$ is the product of a sum $3t + (x + 1)$ and a difference $3t - (x + 1)$.)

$$(3t + x + 1)(3t - x - 1) = [3t + (x+1)][3t - (x+1)]$$
$$= 9t^2 - (x+1)^2$$
$$= 9t^2 - x^2 - 2x - 1$$

Use a special product of binomials to find each product.

1. $(x + y - z)^2$

$x^2 + 2xy - 2xz + y^2 - 2yz + z^2$

2. $(r + s + 5)^2$

$r^2 + 2rs + s^2 + 10r + 10s + 25$

3. $(b - 3 + d)^2$

$b^2 - 6b + 2bd + 9 - 6d + d^2$

4. $(k - m - 2)^2$

$k^2 - 2km + m^2 - 4k + 4m + 4$

5. $(x + 1 + 2b)(x + 1 - 2b)$

$x^2 + 2x + 1 - 4b^2$

6. $(y - 2 + x)(y - 2 - x)$

$y^2 - 4y + 4 - x^2$

7. $(5 + b - x)(5 + b + x)$

$25 + 10b + b^2 - x^2$

8. $(j - 5 - f)(j + 5 + f)$

$j^2 - f^2 - 10f - 25$

9. $[(x + y) + (z + w)][(x + y) - (z + w)]$

$x^2 + 2xy + y^2 - z^2 - 2zw - w^2$

10. $(2a + 1 + 3b - c)(2a + 1 - 3b + c)$

$4a^2 + 4a + 1 - 9b^2 + 6bc - c^2$